Chapter 13
RADIATION HEAT TRANSFER

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Objectives

• Define view factor, and understand its importance in radiation heat transfer calculations

• Develop view factor relations, and calculate the unknown view factors in an enclosure by using these relations

• Calculate radiation heat transfer between black surfaces

• Determine radiation heat transfer between diffuse and gray surfaces in an enclosure using the concept of radiosity

• Obtain relations for net rate of radiation heat transfer between the surfaces of a two-zone enclosure, including two large parallel plates, two long concentric cylinders, and two concentric spheres

• Quantify the effect of radiation shields on the reduction of radiation heat transfer between two surfaces, and become aware of the importance of radiation effect in temperature measurements
**THE VIEW FACTOR**

**View factor** is a purely geometric quantity and is independent of the surface properties and temperature. It is also called the *shape factor*, *configuration factor*, and *angle factor*.

The view factor based on the assumption that the surfaces are diffuse emitters and diffuse reflectors is called the *diffuse view factor*, and the view factor based on the assumption that the surfaces are diffuse emitters but specular reflectors is called the *specular view factor*.

\[ F_{ij} \text{ the fraction of the radiation leaving surface } i \text{ that strikes surface } j \text{ directly} \]

The view factor ranges between 0 and 1. **FIGURE 13-1**

Radiation heat exchange between surfaces depends on the *orientation* of the surfaces relative to each other, and this dependence on orientation is accounted for by the view factor.
To develop a general expression for the view factor, consider two differential surfaces \( dA_1 \) and \( dA_2 \) on two arbitrarily oriented surfaces \( A_1 \) and \( A_2 \), respectively, as shown in Fig. 13–2. The distance between \( dA_1 \) and \( dA_2 \) is \( r \), and the angles between the normals of the surfaces and the line that connects \( dA_1 \) and \( dA_2 \) are \( \theta_1 \) and \( \theta_2 \), respectively. Surface 1 emits and reflects radiation diffusely in all directions with a constant intensity of \( I_1 \), and the solid angle subtended by \( dA_2 \) when viewed by \( dA_1 \) is \( d\omega_{21} \).

The rate at which radiation leaves \( dA_1 \) in the direction of \( \theta_1 \) is \( I_1 \cos \theta_1 dA_1 \). Noting that \( d\omega_{21} = dA_2 \cos \theta_2 / r^2 \), the portion of this radiation that strikes \( dA_2 \) is

\[
\dot{Q}_{dA_1 \rightarrow dA_2} = I_1 \cos \theta_1 dA_1 d\omega_{21} = I_1 \cos \theta_1 dA_1 \frac{dA_2 \cos \theta_2}{r^2}
\]

(13–1)

The total rate at which radiation leaves \( dA_1 \) (via emission and reflection) in all directions is the radiosity (which is \( J_1 = \pi I_1 \)) times the surface area,

\[
\dot{Q}_{dA_1} = J_1 dA_1 = \pi I_1 dA_1
\]

(13–2)

**FIGURE 13–2**
Geometry for the determination of the view factor between two surfaces.
Then the *differential view factor* \( dF_{A_1 \rightarrow A_2} \), which is the fraction of radiation leaving \( A_1 \) that strikes \( A_2 \) directly, becomes

\[
dF_{A_1 \rightarrow A_2} = \frac{\dot{Q}_{A_1 \rightarrow A_2}}{\dot{Q}_{A_1}} = \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_2
\]  

(13–3)

The differential view factor \( dF_{A_1 \rightarrow A_2} \) can be determined from Eq. 13–3 by interchanging the subscripts 1 and 2.

The view factor from a differential area \( dA_1 \) to a finite area \( A_2 \) can be determined from the fact that the fraction of radiation leaving \( dA_1 \) that strikes \( A_2 \) is the sum of the fractions of radiation striking the differential areas \( dA_2 \). Therefore, the view factor \( F_{A_1 \rightarrow A_2} \) is determined by integrating \( dF_{A_1 \rightarrow A_2} \) over \( A_2 \),

\[
F_{A_1 \rightarrow A_2} = \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_2
\]  

(13–4)

The total rate at which radiation leaves the entire \( A_1 \) (via emission and reflection) in all directions is

\[
\dot{Q}_{A_1} = J_1 A_1 - \pi I_1 A_1
\]  

(13–5)

The portion of this radiation that strikes \( dA_2 \) is determined by considering the radiation that leaves \( dA_1 \) and strikes \( dA_2 \) (given by Eq. 13–1), and integrating it over \( A_1 \),

\[
\dot{Q}_{A_1 \rightarrow A_2} = \int_{A_1} \dot{Q}_{A_1 \rightarrow A_2} = \int_{A_1} \frac{I_1 \cos \theta_1 \cos \theta_2}{r^2} dA_2
\]  

(13–6)
Integration of this relation over \( A_2 \) gives the radiation that strikes the entire \( A_2 \),

\[
\hat{Q}_{A_1 \rightarrow A_2} = \int_{A_2} \hat{Q}_{A_1 \rightarrow A_2} = \int_{A_2} \int_{A_1} \frac{I_1 \cos \theta_1 \cos \theta_2}{r^2} dA_1 \, dA_2 \quad (13-7)
\]

Dividing this by the total radiation leaving \( A_1 \) (from Eq. 13–5) gives the fraction of radiation leaving \( A_1 \) that strikes \( A_2 \), which is the view factor \( F_{A_1 \rightarrow A_2} \) (or \( F_{12} \) for short),

\[
F_{12} = F_{A_1 \rightarrow A_2} = \frac{\hat{Q}_{A_1 \rightarrow A_2}}{\hat{Q}_{A_1}} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_1 \, dA_2 \quad (13-8)
\]

The view factor \( F_{A_2 \rightarrow A_1} \) is readily determined from Eq. 13–8 by interchanging the subscripts 1 and 2,

\[
F_{21} = F_{A_2 \rightarrow A_1} = \frac{\hat{Q}_{A_2 \rightarrow A_1}}{\hat{Q}_{A_2}} = \frac{1}{A_2} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_2}{\pi r^2} dA_1 \, dA_2 \quad (13-9)
\]

Note that \( I_1 \) is constant but \( r, \theta_1, \) and \( \theta_2 \) are variables. Also, integrations can be performed in any order since the integration limits are constants. These relations confirm that the view factor between two surfaces depends on their relative orientation and the distance between them.

Combining Eqs. 13–8 and 13–9 after multiplying the former by \( A_1 \) and the latter by \( A_2 \) gives

\[
A_1 F_{12} = A_2 F_{21} \quad (13-10)
\]

which is known as the reciprocity relation for view factors. It allows the calculation of a view factor from a knowledge of the other.
\[ F_{i \rightarrow i} = \text{the fraction of radiation leaving surface } i \text{ that strikes itself directly} \]

**FIGURE 13–3**
The view factor from a surface to itself is zero for plane or convex surfaces and nonzero for concave surfaces.

**FIGURE 13–4**
In a geometry that consists of two concentric spheres, the view factor \[ F_{1 \rightarrow 2} = 1 \] since the entire radiation leaving the surface of the smaller sphere is intercepted by the larger sphere.
The view factor has proven to be very useful in radiation analysis because it allows us to express the fraction of radiation leaving a surface that strikes another surface in terms of the orientation of these two surfaces relative to each other.

The underlying assumption in this process is that the radiation a surface receives from a source is directly proportional to the angle the surface subtends when viewed from the source.

This would be the case only if the radiation coming off the source is uniform in all directions throughout its surface and the medium between the surfaces does not absorb, emit, or scatter radiation.

That is, it is the case when the surfaces are isothermal and diffuse emitters and reflectors and the surfaces are separated by a nonparticipating medium such as a vacuum or air.

View factors for hundreds of common geometries are evaluated and the results are given in analytical, graphical, and tabular form.
### TABLE 13–1

View factor expressions for some common geometries of finite size (3-D)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aligned parallel rectangles</td>
<td>$\bar{X} = X/L$ and $\bar{Y} = Y/L$</td>
</tr>
<tr>
<td></td>
<td>$F_{i \rightarrow j} = \frac{2}{\pi \bar{X} \bar{Y}} \left{ \ln \left[ \frac{(1 + \bar{X}^2)(1 + \bar{Y}^2)}{1 + \bar{X}^2 + \bar{Y}^2} \right]^{1/2} + \bar{X}(1 + \bar{Y}^2)^{1/2} \tan^{-1} \frac{\bar{X}}{(1 + \bar{Y}^2)^{1/2}} + \bar{Y}(1 + \bar{X}^2)^{1/2} \tan^{-1} \frac{\bar{Y}}{(1 + \bar{X}^2)^{1/2}} - \bar{X} \tan^{-1} \bar{X} - \bar{Y} \tan^{-1} \bar{Y} \right}$</td>
</tr>
<tr>
<td>Coaxial parallel disks</td>
<td>$R_i = r_i/L$ and $R_j = r_j/L$</td>
</tr>
<tr>
<td></td>
<td>$S = 1 + \frac{1 + R_j^2}{R_i^2}$</td>
</tr>
<tr>
<td></td>
<td>$F_{i \rightarrow j} = \frac{1}{2} \left{ S - \left[ S^2 - 4 \left( \frac{r_j}{r_i} \right)^2 \right]^{1/2} \right}$</td>
</tr>
<tr>
<td></td>
<td>For $r_i = r_j = r$ and $R = rl$: $F_{i \rightarrow j} = F_{j \rightarrow i} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2}$</td>
</tr>
<tr>
<td>Perpendicular rectangles</td>
<td>$H = Z/X$ and $W = Y/X$</td>
</tr>
<tr>
<td>with a common edge</td>
<td>$F_{i \rightarrow j} = \frac{1}{\pi W} \left( W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H} - (H^2 + W^2)^{1/2} \tan^{-1} \frac{1}{(H^2 + W^2)^{1/2}} \right.$</td>
</tr>
<tr>
<td></td>
<td>$+ \frac{1}{4} \ln \left{ \frac{(1 + W^2)(1 + H^2)}{1 + W^2 + H^2} \left[ \frac{W^2(1 + W^2 + H^2)}{(1 + W^2)(W^2 + H^2)} \right]^{1/2} \right}$</td>
</tr>
<tr>
<td></td>
<td>$\times \left{ \frac{H^2(1 + H^2 + W^2)}{(1 + H^2)(H^2 + W^2)} \left[ H^2 \right] \right}$</td>
</tr>
</tbody>
</table>
## TABLE 13–2

View factor expressions for some infinitely long (2-D) geometries

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel plates with midlines connected by perpendicular line</td>
<td>$W_i = w_i / L$ and $W_j = w_j / L$.</td>
</tr>
<tr>
<td></td>
<td>$F_{i\rightarrow j} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - (W_j - W_i)^2 + 4}{}^{1/2}}{2W_i}$</td>
</tr>
<tr>
<td>Inclined plates of equal width and with a common edge</td>
<td>$F_{i\rightarrow j} = 1 - \sin \frac{1}{2} \alpha$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Perpendicular plates with a common edge</td>
<td>$F_{i\rightarrow j} = \frac{1}{2} \left[ \frac{w_j}{w_i} - \left[ 1 + \left( \frac{w_j}{w_i} \right)^2 \right]^{1/2} \right]$</td>
</tr>
</tbody>
</table>
## Table 13-2

View factor expressions for some infinitely long (2-D) geometries

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-sided enclosure</td>
<td>$F_{i \rightarrow j} = \frac{w_i + w_j - w_k}{2w_i}$</td>
</tr>
<tr>
<td>Infinite plane and row of cylinders</td>
<td>$F_{i \rightarrow j} = 1 - \left[ 1 - \left( \frac{D}{s} \right)^2 \right]^{1/2}$ $+$ $\frac{D}{s} \tan^{-1}\left( \frac{s^2 - D^2}{D^2} \right)^{1/2}$</td>
</tr>
</tbody>
</table>
View factor between two aligned parallel rectangles of equal size.
View factor between two perpendicular rectangles with a common edge.
View factor between two coaxial parallel disks.
View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder; (b) outer cylinder to itself.
View Factor Relations

Radiation analysis on an enclosure consisting of $N$ surfaces requires the evaluation of $N^2$ view factors.

Once a sufficient number of view factors are available, the rest of them can be determined by utilizing some fundamental relations for view factors.

1 The Reciprocity Relation

$$F_{j \rightarrow i} = F_{i \rightarrow j} \quad \text{when} \quad A_i = A_j$$

$$F_{j \rightarrow i} \neq F_{i \rightarrow j} \quad \text{when} \quad A_i \neq A_j$$

$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i} \quad \text{reciprocity relation (rule)}$$
The sum of the view factors from surface $i$ of an enclosure to all surfaces of the enclosure, including to itself, must equal unity.

\[ \sum_{j=1}^{N} F_{i \rightarrow j} = 1 \]

\[ \sum_{j=1}^{3} F_{1 \rightarrow j} = F_{1 \rightarrow 1} + F_{1 \rightarrow 2} + F_{1 \rightarrow 3} = 1 \]

The total number of view factors that need to be evaluated directly for an $N$-surface enclosure is

\[ N^2 - \left[ N + \frac{1}{2} N(N - 1) \right] = \frac{1}{2} N(N - 1) \]

The remaining view factors can be determined from the equations that are obtained by applying the reciprocity and the summation rules.
3 The Superposition Rule

The view factor from a surface $i$ to a surface $j$ is equal to the sum of the view factors from surface $i$ to the parts of surface $j$.

$$ F_{1 \rightarrow (2, 3)} = F_{1 \rightarrow 2} + F_{1 \rightarrow 3} $$

multiply by $A_1$

$$ A_1 F_{1 \rightarrow (2, 3)} = A_1 F_{1 \rightarrow 2} + A_1 F_{1 \rightarrow 3} $$

apply the reciprocity relation

$$ (A_2 + A_3) F_{(2, 3) \rightarrow 1} = A_2 F_{2 \rightarrow 1} + A_3 F_{3 \rightarrow 1} $$

$$ F_{(2, 3) \rightarrow 1} = \frac{A_2 F_{2 \rightarrow 1} + A_3 F_{3 \rightarrow 1}}{A_2 + A_3} $$

FIGURE 13–11

The view factor from a surface to a composite surface is equal to the sum of the view factors from the surface to the parts of the composite surface.
4 The Symmetry Rule

Two (or more) surfaces that possess symmetry about a third surface will have identical view factors from that surface.

If the surfaces $j$ and $k$ are symmetric about the surface $i$ then

\[ F_{i \rightarrow j} = F_{i \rightarrow k} \quad \text{and} \quad F_{j \rightarrow i} = F_{k \rightarrow i} \]

**FIGURE 13–13**

Two surfaces that are symmetric about a third surface will have the same view factor from the third surface.
View Factors between Infinitely Long Surfaces: The Crossed-Strings Method

Channels and ducts that are very long in one direction relative to the other directions can be considered to be two-dimensional.

These geometries can be modeled as being infinitely long, and the view factor between their surfaces can be determined by simple crossed-strings method.

\[
F_{1 \rightarrow 2} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}
\]

\[
F_{i \rightarrow j} = \frac{\sum (\text{Crossed strings}) - \sum (\text{Uncrossed strings})}{2 \times (\text{String on surface } i)}
\]

FIGURE 13–16

Determination of the view factor \(F_{1 \rightarrow 2}\) by the application of the crossed-strings method.
When the surfaces involved can be approximated as blackbodies because of the absence of reflection, the net rate of radiation heat transfer from surface 1 to surface 2 is

\[ \dot{Q}_{1 \rightarrow 2} = \left( \text{Radiation leaving the entire surface 1 that strikes surface 2} \right) - \left( \text{Radiation leaving the entire surface 2 that strikes surface 1} \right) \]

\[ = A_1 E_{b1} F_{1 \rightarrow 2} - A_2 E_{b2} F_{2 \rightarrow 1} \tag{W} \]

The net radiation heat transfer from any surface \( i \) of an \( N \) surface enclosure is

\[ \dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i \rightarrow j} = \sum_{j=1}^{N} A_i E_{b} F_{i \rightarrow j} \sigma(T_i^4 - T_j^4) \tag{W} \]

A negative value for \( \dot{Q}_{1 \rightarrow 2} \) indicates that net radiation heat transfer is from surface 2 to surface 1.
RADIATION HEAT TRANSFER: DIFFUSE, GRAY SURFACES

• Most enclosures encountered in practice involve nonblack surfaces, which allow multiple reflections to occur.
• Radiation analysis of such enclosures becomes very complicated unless some simplifying assumptions are made.
• It is common to assume the surfaces of an enclosure to be opaque, diffuse, and gray.
• Also, each surface of the enclosure is isothermal, and both the incoming and outgoing radiation are uniform over each surface.
Radiosity

Radiosity $J$: The total radiation energy leaving a surface per unit time and per unit area.

For a surface $i$ that is gray and opaque ($\varepsilon_i = \alpha_i$ and $\alpha_i + \rho_i = 1$)

$$J_i = \left( \text{Radiation emitted by surface } i \right) + \left( \text{Radiation reflected by surface } i \right)$$

$$= \varepsilon_i E_{bi} + \rho_i G_i$$

$$= \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i \quad (\text{W/m}^2)$$

For a blackbody $\varepsilon = 1$

$$J_i = E_{bi} = \sigma T_i^4 \quad \text{(blackbody)}$$

The radiosity of a blackbody is equal to its emissive power since radiation coming from a blackbody is due to emission only.

**FIGURE 13–20**
Radiosity represents the sum of the radiation energy emitted and reflected by a surface.
Net Radiation Heat Transfer to or from a Surface

The net rate of radiation heat transfer from a surface $i$

\[ \dot{Q}_i = \left( \text{Radiation leaving entire surface } i \right) - \left( \text{Radiation incident on entire surface } i \right) \]

\[ = A_i (J_i - G_i) \quad \text{(W)} \]

\[ \dot{Q}_i = A_i \left( J_i - \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i} \right) = \frac{A_i \varepsilon_i}{1 - \varepsilon_i} (E_{bi} - J_i) \quad \text{(W)} \]

\[ \dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \quad \text{(W)} \]

\[ R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i} \quad \text{surface resistance to radiation.} \]

The surface resistance to radiation for a blackbody is zero since $\varepsilon_i = 1$ and $J_i = E_{bi}$.

Reradiating surface: Some surfaces are modeled as being adiabatic since their back sides are well insulated and the net heat transfer through them is zero.

\[ J_i = E_{bi} = \sigma T_i^4 \quad \text{(W/m}^2\text{)} \]

**FIGURE 13–21**

Electrical analogy of surface resistance to radiation.
Net Radiation Heat Transfer between Any Two Surfaces

The net rate of radiation heat transfer from surface \( i \) to surface \( j \) is

\[
\dot{Q}_{i \rightarrow j} = \left( \text{Radiation leaving the entire surface } i \right) - \left( \text{Radiation leaving the entire surface } j \right) - \left( \text{that strikes surface } j \right) + \left( \text{that strikes surface } i \right)
\]

\[
= A_i J_i F_{i \rightarrow j} - A_j J_j F_{j \rightarrow i}
\]  \[(W)\]

Apply the reciprocity relation

\[
A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i}
\]

\[
\dot{Q}_{i \rightarrow j} = A_i F_{i \rightarrow j} (J_i - J_j)
\]  \[(W)\]

\[
\dot{Q}_{i \rightarrow j} = \frac{J_i - J_j}{R_{i \rightarrow j}}
\]  \[(W)\]

\[
R_{i \rightarrow j} = \frac{1}{A_i F_{i \rightarrow j}}
\]

space resistance to radiation

**FIGURE 13–22**
Electrical analogy of space resistance to radiation.
In an $N$-surface enclosure, the conservation of energy principle requires that the net heat transfer from surface $i$ be equal to the sum of the net heat transfers from surface $i$ to each of the $N$ surfaces of the enclosure.

\[
\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i \rightarrow j} = \sum_{j=1}^{N} A_i F_{i \rightarrow j} (J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i \rightarrow j}} \quad (W)
\]

**FIGURE 13–23**

Network representation of net radiation heat transfer from surface $i$ to the remaining surfaces of an $N$-surface enclosure.

The net radiation flow from a surface through its surface resistance is equal to the sum of the radiation flows from that surface to all other surfaces through the corresponding space resistances.
Methods of Solving Radiation Problems

In the radiation analysis of an enclosure, either the temperature or the net rate of heat transfer must be given for each of the surfaces to obtain a unique solution for the unknown surface temperatures and heat transfer rates.

\[
\dot{Q}_i = A_i \sum_{j=1}^{N} F_{i \rightarrow j} (J_i - J_j)
\]

\[
\sigma T_i^4 = J_i + \frac{1 - \varepsilon_i}{\varepsilon_i} \sum_{j=1}^{N} F_{i \rightarrow j} (J_i - J_j)
\]

The equations above give \( N \) linear algebraic equations for the determination of the \( N \) unknown radiosities for an \( N \)-surface enclosure. Once the radiosities \( J_1, J_2, \ldots, J_N \) are available, the unknown heat transfer rates and the unknown surface temperatures can be determined from the above equations.

**Direct method**: Based on using the above procedure. This method is suitable when there are a large number of surfaces.

**Network method**: Based on the electrical network analogy. Draw a surface resistance associated with each surface of an enclosure and connect them with space resistances. Then solve the radiation problem by treating it as an electrical network problem. The network method is not practical for enclosures with more than three or four surfaces.
Radiation Heat Transfer in Two-Surface Enclosures

This important result is applicable to any two gray, diffuse, and opaque surfaces that form an enclosure.

\[
\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2
\]

\[
\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2} = \dot{Q}_1 = -\dot{Q}_2
\]

\[
\dot{Q}_{12} = \sigma(T_1^4 - T_2^4) \left( \frac{1 - \varepsilon_1}{A_1 \varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \varepsilon_2}{A_2 \varepsilon_2} \right) \quad \text{(W)}
\]

**FIGURE 13–24**
Schematic of a two-surface enclosure and the radiation network associated with it.
### TABLE 13–3
Radiation heat transfer relations for some familiar two-surface arrangements.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small object in a large cavity</td>
<td>$A_1/A_2 = 0$  \quad $F_{12} = 1$  \quad $\dot{Q}_{12} = A_1C\varepsilon_1(T_1^4 - T_2^4)$ \quad (13–37)</td>
</tr>
<tr>
<td>Infinitely large parallel plates</td>
<td>$A_1 = A_2 = A$  \quad $F_{12} = 1$  \quad $\dot{Q}_{12} = \frac{A \sigma (T_1^4 - T_2^4)}{\varepsilon_1 + \frac{1}{\varepsilon_2} - 1}$ \quad (13–38)</td>
</tr>
<tr>
<td>Infinitely long concentric cylinders</td>
<td>$\frac{A_1}{A_2} = \frac{r_1}{r_2}$  \quad $F_{12} = 1$  \quad $\dot{Q}_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left(\frac{r_1}{r_2}\right)}$ \quad (13–39)</td>
</tr>
<tr>
<td>Concentric spheres</td>
<td>$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^3$  \quad $F_{12} = 1$  \quad $\dot{Q}_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left(\frac{r_1}{r_2}\right)^2}$ \quad (13–40)</td>
</tr>
</tbody>
</table>
Radiation Heat Transfer in Three-Surface Enclosures

When \( Q_i \) is specified at surface \( i \) instead of the temperature, the term \((E_{bi} - J_i)/R_i\) should be replaced by the specified \( Q_i \).

The algebraic sum of the currents (net radiation heat transfer) at each node must equal zero.

\[
\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0
\]
\[
\frac{J_1 - J_2}{R_{12}} + \frac{E_{b2} - J_2}{R_2} + \frac{J_3 - J_2}{R_{23}} = 0
\]
\[
\frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} + \frac{E_{b3} - J_3}{R_3} = 0
\]

These equations are to be solved for \( J_1, J_2, \) and \( J_3 \).

Draw a surface resistance associated with each of the three surfaces and connect them with space resistances.

Schematic of a three-surface enclosure and the radiation network associated with it.
Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high-reflectivity (low-emissivity) sheet of material between the two surfaces.

Such highly reflective thin plates or shells are called radiation shields.

Multilayer radiation shields constructed of about 20 sheets per cm thickness separated by evacuated space are commonly used in cryogenic and space applications.

Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect when the temperature sensor is exposed to surfaces that are much hotter or colder than the fluid itself.

*The role of the radiation shield is to reduce the rate of radiation heat transfer by placing additional resistances in the path of radiation heat flow.*

The lower the emissivity of the shield, the higher the resistance.
Radiation heat transfer between two large parallel plates with one shield

\[
\dot{Q}_{12, \text{one shield}} = \frac{E_{b1} - E_{b2}}{\frac{1 - \varepsilon_1}{A_1 \varepsilon_1} + \frac{1}{A_1 F_{13}} + \frac{1 - \varepsilon_{3,1}}{A_3 \varepsilon_{3,1}} + \frac{1 - \varepsilon_{3,2}}{A_3 \varepsilon_{3,2}} + \frac{1}{A_3 F_{32}} + \frac{1 - \varepsilon_2}{A_2 \varepsilon_2}}
\]

The radiation shield placed between two parallel plates and the radiation network associated with it.
\[ \dot{Q}_{12, \text{one shield}} = \frac{A \sigma (T_1^4 - T_2^4)}{\left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \right) + \left( \frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1 \right)} \]

\[ \dot{Q}_{12, N \text{ shields}} = \frac{\Lambda \sigma (T_1^4 - T_2^4)}{\left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \right) + \left( \frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1 \right) + \cdots + \left( \frac{1}{\varepsilon_{N,1}} + \frac{1}{\varepsilon_{N,2}} - 1 \right)} \]

\[ \dot{Q}_{12, N \text{ shields}} = \frac{A \sigma (T_1^4 - T_2^4)}{(N + 1) \left( \frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1 \right)} = \frac{1}{N + 1} \dot{Q}_{12, \text{no shield}} \]

If the emissivities of all surfaces are equal
Radiation Effect on Temperature Measurements

FIGURE 13–31
A thermometer used to measure the temperature of a fluid in a channel.

The last term in the equation is due to the *radiation effect* and represents the *radiation correction*. The radiation correction term is most significant when the convection heat transfer coefficient is small and the emissivity of the surface of the sensor is large. Therefore, the sensor should be coated with a material of high reflectivity (low emissivity) to reduce the radiation effect.
So far we considered radiation heat transfer between surfaces separated by a medium that does not emit, absorb, or scatter radiation—a nonparticipating medium that is completely transparent to thermal radiation.

Gases with asymmetric molecules such as $\text{H}_2\text{O}$, $\text{CO}_2$, $\text{CO}$, $\text{SO}_2$, and hydrocarbons $\text{H}_m\text{C}_n$ may participate in the radiation process by absorption at moderate temperatures, and by absorption and emission at high temperatures such as those encountered in combustion chambers. Therefore, air or any other medium that contains such gases with asymmetric molecules at sufficient concentrations must be treated as a participating medium in radiation calculations.

Combustion gases in a furnace or a combustion chamber, for example, contain sufficient amounts of $\text{H}_2\text{O}$ and $\text{CO}_2$, and thus the emission and absorption of gases in furnaces must be taken into consideration.
We consider the emission and absorption of radiation by H₂O and CO₂ only since they are the participating gases most commonly encountered in practice.

The presence of a participating medium complicates the radiation analysis considerably for several reasons:

- A participating medium emits and absorbs radiation throughout its entire volume. That is, gaseous radiation is a *volumetric phenomena*, and thus it depends on the size and shape of the body. This is the case even if the temperature is uniform throughout the medium.

- Gases emit and absorb radiation at a number of narrow wavelength bands. This is in contrast to solids, which emit and absorb radiation over the entire spectrum. Therefore, the gray assumption may not always be appropriate for a gas even when the surrounding surfaces are gray.

- The emission and absorption characteristics of the constituents of a gas mixture also depends on the temperature, pressure, and composition of the gas mixture. Therefore, the presence of other participating gases affects the radiation characteristics of a particular gas.
Radiation Properties of a Participating Medium

Consider a participating medium of thickness $L$. A spectral radiation beam of intensity $I_{\lambda,0}$ is incident on the medium, which is attenuated as it propagates due to absorption. The decrease in the intensity of radiation as it passes through a layer of thickness $dx$ is proportional to the intensity itself and the thickness $dx$. This is known as Beer’s law, and is expressed as (Fig. 13–34)

\[
dI_\lambda(x) = -\kappa_\lambda I_\lambda(x)dx
\]

(13–47)

where the constant of proportionality $\kappa_\lambda$ is the spectral absorption coefficient of the medium whose unit is $\text{m}^{-1}$ (from the requirement of dimensional homogeneity). This is just like the amount of interest earned by a bank account during a time interval being proportional to the amount of money in the account and the time interval, with the interest rate being the constant of proportionality.

Separating the variables and integrating from $x = 0$ to $x = L$ gives

\[
\frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L}
\]

(13–48)

where we have assumed the absorptivity of the medium to be independent of $x$. Note that radiation intensity decays exponentially in accordance with Beer’s law.

The spectral transmissivity of a medium can be defined as the ratio of the intensity of radiation leaving the medium to that entering the medium. That is,

\[
\tau_\lambda = \frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L}
\]

(13–49)

Note that $\tau_\lambda = 1$ when no radiation is absorbed and thus radiation intensity remains constant. Also, the spectral transmissivity of a medium represents the fraction of radiation transmitted by the medium at a given wavelength.
Radiation passing through a nonscattering (and thus nonreflecting) medium is either absorbed or transmitted. Therefore $\alpha_\lambda + \tau_\lambda = 1$, and the **spectral absorptivity** of a medium of thickness $L$ is

$$\alpha_\lambda = 1 - \tau_\lambda = 1 - e^{-\kappa_\lambda L} \quad (13-50)$$

From Kirchoff’s law, the **spectral emissivity** of the medium is

$$\varepsilon_\lambda = \alpha_\lambda = 1 - e^{-\kappa_\lambda L} \quad (13-51)$$

Note that the spectral absorptivity, transmissivity, and emissivity of a medium are dimensionless quantities, with values less than or equal to 1. The spectral absorption coefficient of a medium (and thus $\varepsilon_\lambda$, $\alpha_\lambda$, and $\tau_\lambda$), in general, vary with wavelength, temperature, pressure, and composition.

For an **optically thick** medium (a medium with a large value of $\kappa_\lambda L$), Eq. 13-51 gives $\varepsilon_\lambda \sim \alpha_\lambda \sim 1$. For $\kappa_\lambda L = 5$, for example, $\varepsilon_\lambda = \alpha_\lambda = 0.993$. Therefore, an optically thick medium emits like a blackbody at the given wavelength. As a result, an optically thick absorbing-emitting medium with no significant scattering at a given temperature $T_g$ can be viewed as a “black surface” at $T_g$ since it will absorb essentially all the radiation passing through it, and it will emit the maximum possible radiation that can be emitted by a surface at $T_g$, which is $E_{b\lambda}(T_g)$. 
Emissivity and Absorptivity of Gases and Gas Mixtures

The various peaks and dips in the figure together with discontinuities show clearly the band nature of absorption and the strong nongray characteristics. The shape and the width of these absorption bands vary with temperature and pressure, but the magnitude of absorptivity also varies with the thickness of the gas layer. Therefore, absorptivity values without specified thickness and pressure are meaningless.

**FIGURE 13–35**
Spectral absorptivity of CO₂ at 830 K and 10 atm for a path length of 38.8 cm
The emissivity of H$_2$O vapor in a mixture of nonparticipating gases is plotted in Figure 13–36a for a total pressure of $P = 1$ atm as a function of gas temperature $T_g$ for a range of values for $P_w L$, where $P_w$ is the partial pressure of water vapor and $L$ is the mean distance traveled by the radiation beam. Emissivity at a total pressure $P$ other than $P = 1$ atm is determined by multiplying the emissivity value at 1 atm by a pressure correction factor $C_w$ obtained from Figure 13–37a for water vapor. That is,

$$\varepsilon_w = C_w \varepsilon_{w,1\text{ atm}} \quad (13–52)$$

Note that $C_w = 1$ for $P = 1$ atm and thus $(P_w + P)/2 \approx 0.5$ (a very low concentration of water vapor is used in the preparation of the emissivity chart in Fig. 13–36a and thus $P_w$ is very low). Emissivity values are presented in a similar manner for a mixture of CO$_2$ and nonparticipating gases in Figs. 13–36b and 13–37b.

Now the question that comes to mind is what will happen if the CO$_2$ and H$_2$O gases exist together in a mixture with nonparticipating gases. The emissivity of each participating gas can still be determined as explained above using its partial pressure, but the effective emissivity of the mixture cannot be determined by simply adding the emissivities of individual gases (although this would be the case if different gases emitted at different wavelengths). Instead, it should be determined from

$$\varepsilon_g = \varepsilon_c + \varepsilon_w - \Delta\varepsilon$$

$$= C_c \varepsilon_{c,1\text{ atm}} + C_w \varepsilon_{w,1\text{ atm}} - \Delta\varepsilon \quad (13–53)$$

where $\Delta\varepsilon$ is the emissivity correction factor, which accounts for the overlap of emission bands. For a gas mixture that contains both CO$_2$ and H$_2$O gases, $\Delta\varepsilon$ is plotted in Figure 13–38.
The emissivity of a gas also depends on the *mean length* an emitted radiation beam travels in the gas before reaching a bounding surface, and thus the shape and the size of the gas body involved. During their experiments in the 1930s, Hottel and his coworkers considered the emission of radiation from a hemispherical gas body to a small surface element located at the center of the base of the hemisphere. Therefore, the given charts represent emissivity data for the emission of radiation from a hemispherical gas body of radius $L$ toward the center of the base of the hemisphere. It is certainly desirable to extend the reported emissivity data to gas bodies of other geometries, and this is done by introducing the concept of *mean beam length* $L$, which represents the radius of an equivalent hemisphere. The mean beam lengths for various gas geometries are listed in Table 13–4. More extensive lists are available in the literature.

Following a procedure recommended by Hottel, the absorptivity of a gas that contains $CO_2$ and $H_2O$ gases for radiation emitted by a source at temperature $T_s$ can be determined similarly from

$$\alpha_g = \alpha_c + \alpha_w - \Delta \alpha$$

(13-54)

where $\Delta \alpha = \Delta \varepsilon$ and is determined from Figure 13–38 at the source temperature $T_s$. The absorptivities of $CO_2$ and $H_2O$ can be determined from the emissivity charts (Figs. 12–36 and 12–37) as

$CO_2$: \hspace{1cm} \alpha_c = C_c \times (T_g/T_s)^{0.65} \times \varepsilon_c(T_s, P_c LT_s / T_g)

(13-55)$

and

$H_2O$: \hspace{1cm} \alpha_w = C_w \times (T_g/T_s)^{0.45} \times \varepsilon_w(T_s, P_w LT_s / T_g)

(13-56)$
The notation indicates that the emissivities should be evaluated using $T_s$ instead of $T_g$ (both in K or R), $P_c LT_s/T_g$ instead of $P_c L$, and $P_w LT_s/T_g$ instead of $P_w L$. Note that the absorptivity of the gas depends on the source temperature $T_s$ as well as the gas temperature $T_g$. Also, $\alpha = \varepsilon$ when $T_s = T_g$, as expected. The pressure correction factors $C_c$ and $C_w$ are evaluated using $P_c L$ and $P_w L$, as in emissivity calculations.

When the total emissivity of a gas $\varepsilon_g$ at temperature $T_g$ is known, the emissive power of the gas (radiation emitted by the gas per unit surface area) can be expressed as $E_g = \varepsilon_g \sigma T_g^4$. Then the rate of radiation energy emitted by a gas to a bounding surface of area $A_s$ becomes

$$\dot{Q}_{g,e} = \varepsilon_g A_s \sigma T_g^4$$  \hspace{1cm} (13–57)

If the bounding surface is black at temperature $T_s$, the surface will emit radiation to the gas at a rate of $A_s \sigma T_s^4$ without reflecting any, and the gas will absorb this radiation at a rate of $\alpha_g A_s \sigma T_s^4$, where $\alpha_g$ is the absorptivity of the gas. Then the net rate of radiation heat transfer between the gas and a black surface surrounding it becomes

**Black enclosure:**

$$\dot{Q}_{\text{net}} = A_s \sigma (\varepsilon_g T_g^4 - \alpha_g T_s^4)$$  \hspace{1cm} (13–58)

If the surface is not black, the analysis becomes more complicated because of the radiation reflected by the surface. But for surfaces that are nearly black with an emissivity $\varepsilon_s > 0.7$, Hottel (1954), recommends this modification,

$$\dot{Q}_{\text{net, gray}} = \frac{\varepsilon_s + 1}{2} \dot{Q}_{\text{net, black}} = \frac{\varepsilon_s + 1}{2} A_s \sigma (\varepsilon_g T_g^4 - \alpha_g T_s^4)$$  \hspace{1cm} (13–59)

The emissivity of wall surfaces of furnaces and combustion chambers are typically greater than 0.7, and thus the relation above provides great convenience for preliminary radiation heat transfer calculations.
**FIGURE 13–36**

Emissivities of H$_2$O and CO$_2$ gases in a mixture of nonparticipating gases at a total pressure of 1 atm for a mean beam length of $L$ (1 m·atm = 3.28 ft·atm)
FIGURE 13–37
Correction factors for the emissivities of H₂O and CO₂ gases at pressures other than 1 atm for use in the relations $\varepsilon_w = C_w \varepsilon_{w,1\text{ atm}}$ and $\varepsilon_c = C_c \varepsilon_{c,1\text{ atm}}$ (1 m-atm = 3.28 ft-atm)
Emissivity correction $\Delta \varepsilon$ for use in $\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta \varepsilon$ when both CO$_2$ and H$_2$O vapor are present in a gas mixture (1 m·tm = 3.28 ft·atm)
<table>
<thead>
<tr>
<th>Gas Volume Geometry</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemisphere of radius $R$ radiating to the center of its base</td>
<td>$R$</td>
</tr>
<tr>
<td>Sphere of diameter $D$ radiating to its surface</td>
<td>$0.65D$</td>
</tr>
<tr>
<td>Infinite circular cylinder of diameter $D$ radiating to curved surface</td>
<td>$0.95D$</td>
</tr>
<tr>
<td>Semi-infinite circular cylinder of diameter $D$ radiating to its base</td>
<td>$0.65D$</td>
</tr>
<tr>
<td>Semi-infinite circular cylinder of diameter $D$ radiating to center of its base</td>
<td>$0.90D$</td>
</tr>
<tr>
<td>Infinite semicircular cylinder of radius $R$ radiating to center of its base</td>
<td>$1.26R$</td>
</tr>
<tr>
<td>Circular cylinder of height equal to diameter $D$ radiating to entire surface</td>
<td>$0.60D$</td>
</tr>
<tr>
<td>Circular cylinder of height equal to diameter $D$ radiating to center of its base</td>
<td>$0.71D$</td>
</tr>
<tr>
<td>Infinite slab of thickness $D$ radiating to either bounding plane</td>
<td>$1.80D$</td>
</tr>
<tr>
<td>Cube of side length $L$ radiating to any face</td>
<td>$0.66L$</td>
</tr>
<tr>
<td>Arbitrary shape of volume $V$ and surface area $A_s$ radiating to surface</td>
<td>$3.6V/A_s$</td>
</tr>
</tbody>
</table>
Summary

• The View Factor
• View Factor Relations
• Radiation Heat Transfer: Black Surfaces
• Radiation Heat Transfer: Diffuse, Gray Surfaces
  ✓ Radiosity
  ✓ Net Radiation Heat Transfer to or from a Surface
  ✓ Net Radiation Heat Transfer between Any Two Surfaces
  ✓ Methods of Solving Radiation Problems
  ✓ Radiation Heat Transfer in Two-Surface Enclosures
  ✓ Radiation Heat Transfer in Three-Surface Enclosures
• Radiation Shields and The Radiation Effects
  ✓ Radiation Effect on Temperature Measurements
• Radiation Exchange with Emitting and Absorbing Gases
  ✓ Radiation Properties of a Participating Medium
  ✓ Emissivity and Absorptivity of Gases and Gas Mixtures